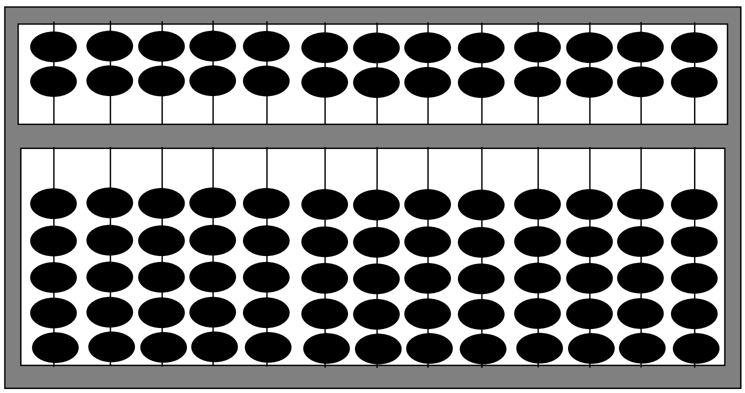
The Abacus:

The Art of Calculation using Beads



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1.0 Introduction

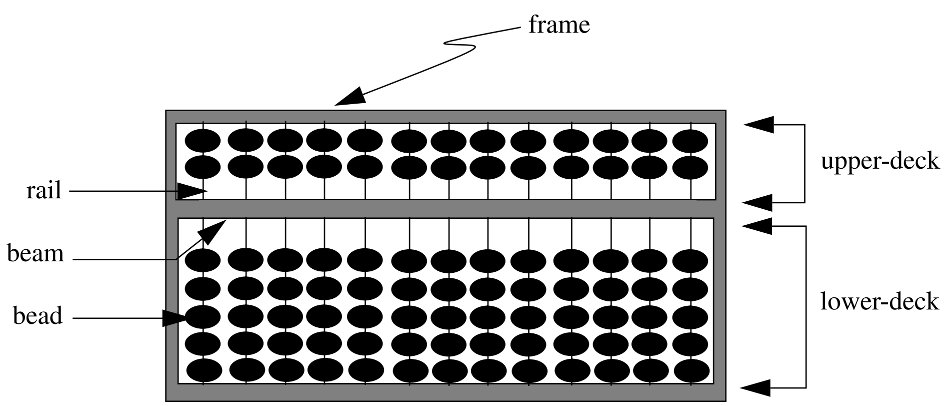
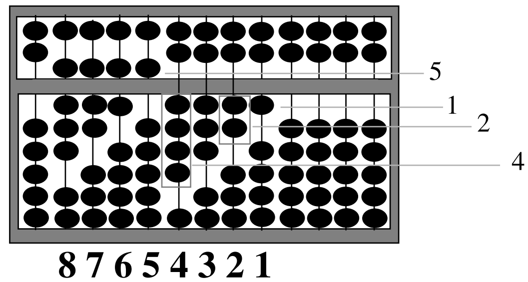


FIGURE 1 The Anatomy of an Abacus

The classic abacus has two decks. Each deck, separated by a beam, has several (normally 13) rods on which are mounted beads. Each rod on the top deck contains 2 beads, and each rod on the bottom deck contains 5 beads. Each bead on the upper deck has a value of five, while each bead on the lower deck has value of one.

Beads are considered counted, when moved *towards* the beam separating the decks.

FIGURE 2 Numeric Representation of Number 87,654,321 on the Abacus



2.0 Addition

Addition on the abacus involves registering the numbers on the beads in the straight-forward left-to-right sequence they are written down in. As long as the digits are placed correctly, and the carry's noted properly, the answer to the operation immediately presents itself right on the abacus. There are 4 approaches to performing additions (or subtractions), each applied to particular situations. Each of these techniques is explained in tabular form in the sections that follow[1].

2.1 Simple Addition

When performing the addition 6+2, one would move 1 bead from the upper deck down (value = 5) and one bead from the lower deck up (value = 1); this represents 6. Moving 2 beads from the lower deck (in the same column) up (value = 1 * 2 beads = 2) would complete the operation. The answer is then obtained by reading resultant bead positions.

TABLE 1 Simple Addition

Original Number	Formula	Bead-Operations Involved	
		lower-deck	upper-deck
0,1,2,3,4,5,6,7,8	plus 1	+1	
0,1,2,5,6 or 7	plus 2	+2	
0,1,5 or 6	plus 3	+3	
0 or 3	plus 4	+4	
0,1,2,3 or 4	plus 5		+1
0,1,2, or 3	plus 6	+1	+1
0,1 or 2	plus 7	+2	+1
0 or 1	plus 8	+3	+1
0	plus 9	+4	+1

1. The tables are used as follows: given *Original Number*, to add/subtract *Formula*, perform *Bead-Operations Involved*.
For example, to add 2 and 7, move down *Original Number* column to a row containing 2 and down *Formula* to a row containing 7 (the 7th row) then move across to read the *Operation Involved*: move 2 bead counters in the lower-deck and 1 bead counter in the upper-deck to represent the sum; the answer then presents itself on the abacus. The "+" symbol, in the *Operations Involved* column represents moving the bead in question, *towards* the rail; the "-" symbol represents moving the bead in question *away* from the rail.

2.2 Combined Adding-up And Taking Off

When the original number registered on a rod is smaller than 5, but will become greater than 5 after the addition, one bead from the upper-deck is moved down (added on to the beam) and one or more beads from the lower deck removed from the beam.

TABLE 2 Combined Adding-up And Taking Off

Original Number	Formula	Bead-Operations Involved lower-deck	upper-deck
4	plus 5 (+5 - 4)	- 4	+1
4 or 3	plus 6 (+5 - 3)	- 3	+1
4,3, or 2	plus 7 (+5 - 2)	- 2	+1
4,3,2 or 1	plus 8 (+5 -1)	- 1	+1

2.3 Combined Taking-off And Place Advancement

When a sum greater than 10 occurs on a certain rod, beads are removed from either or both the upper and lower decks and 1 bead is added to the rod directly to the left. Example: When adding 9 (10-1) to 8, one bead from the lower deck is removed (-1) and one bead from the lower deck on the row directly to the left is added (+10).

TABLE 3 Combined Adding-up And Place Advancement

Original Number	Formula	Bead-Operations Involved		
		lower-deck	upper-deck	lower-deck, previous rod
9	plus 1 (-9 +10)	- 4	-1	+1
8 or 9	plus 2 (-8 +10)	- 3	- 1	+1
7,8 or 6	plus 3 (-7 +10)	- 2	- 1	+1
6,7,8 or 9	plus 4 (-6 +10)	- 1	- 1	+1
5,6,7,8 or 9	plus 5 (-5 +10)		- 1	+1
4or 9	plus 6 (-4 +10)	- 4		+1
3,4,8 or 9	plus 7 (-3 +10)	- 3		+1
2,3,4,7,8 or 9	plus 8 (-2 +10)	- 2		+1
1,2,3,4,6,7,8,9	plus 9 (-1 +10)	- 1		+1

2.4 Combined Adding-up, Taking-off And Place Advancement

There are 4 cases when beads are added to the lower-deck, removed from the upper-deck and one bead added to the adjacent rod.

Example: When adding 7 to 6 (+1-5+10), one bead is added to the lower-deck, one bead removed from the upper-deck and one bead is added to the left rod (lower-deck).

TABLE 4 Combined Adding-up And Taking Off and Place Advancement

Original Number	Formula	Bead-Operations Involved lower-deck	upper-deck	lower-deck, previous rod
5,6,7 or 8	plus 6 (+1-5+10)	+1	-1	+1
5,6 or 7	plus 2 (+2-5+10)	+2	- 1	+1
5 or 6	plus 3 (+3-5+10)	+3	- 1	+1
5	plus 4 (+4-5+10)	+4	- 1	+1

3.0 Subtraction

Subtraction is performed by first registering the minuend and then subtracting, starting from the left, by removing beads form either or both the lower or upper decks. The final bead-positions represent the answer.

3.1 Simple Taking-off

This is achieved by simply taking off one or more beads from the lower deck, or sometimes both. Example: When subtracting 7 (represented by -5-2= -7) from 9, remove 1 bead from the upper-deck (-5) and 2 beads from the lower deck (-2). The remaining 2 beads represent the result.

TABLE 5 Simple Taking-off

Original Number	Formula	Bead-Operations Involved lower-deck	upper-deck
1,2,3,4,5,6,7,8	minus 1	-1	
2,3,4,7,8 or 9	minus 2	-2	
3,4,8 or 9	minus 3	-3	
4 or 9	minus 4	-4	
5,6,7,8 or 9	minus 5		-1
6,7,8 or 9	minus 6	-1	-1
7,8 or 9	minus 7	-2	-1
8 or 9	minus 8	-3	-1
9	minus 9	-4	-1

3.2 Combined Adding-up And Taking Off

When the number of beads in the lower deck is less than the subtracter (the number being subtracted), one or more beads are added in the lower deck and 1 bead is removed from the upper-deck.

Example: When subtracting 4 (+1-5 = -4) from 7 (represented by 1 bead in the upper-deck and 2 beads in the lower deck (less than 4, the subtracter), one bead is added to the lower deck (+1) and 1 bead is removed from the upper-deck (-5) leaving 3 beads, representing the result.

TABLE 6 Combined Adding-up And Taking Off

Original Number	Formula	Bead-Operations Involved lower-deck	upper-deck
5	minus1 (-5 + 4)	+ 4	-1
5 or 6	minus 2 (-5 + 3)	+ 3	-1
5,6 or 7	minus 3 (-5 + 2)	+ 2	-1
5,6,7 or 8	minus 4 (-5 + 1)	+ 1	-1

3.3 Taking-off From A Rod Of Higher Order And Adding-up

When a number on a specific rod is smaller than the subtrahend (4 is the subtrahend when performing 13 - 4; note that in the ones column, the 3 is less than the 4) one bead for the order of tens and one bead from the lower-deck has to be *taken off,* and one bead from the upper-deck is counted.

TABLE 7 Taking-off From A Rod Of Higher Order And Adding-up

Original Number	Formula	Bead-Operations Involved		
		lower-deck, right 1 rod	lower-deck	upper-deck
0	minus 1 (+9 -10)	-1	+ 4	+ 1
0 or 1	minus 2 (+8 -10)	- 1	+ 3	+ 1
0,1 or 2	minus 3 (+7 -10)	- 1	+ 2	+ 1
0,1,2 or 9	minus 4 (+6 -10)	- 1	+ 1	+ 1
0,1,2,3 or 4	minus 5 (+5 -10)	- 1		+ 1
0 or 5	minus 6 (+4 -10)	- 1	+ 4	
0,5 or 6	minus 7 (+3 -10)	- 1	+ 3	
0,1,2,5,6 or 7	minus 8 (+2 -10)	- 1	+ 2	
0,1,2,3,5,6,7,8	minus 9 (+1 -10)	- 1	+ 1	

3.4 Combined Taking-off From A Rod Of Higher Order, Adding-up in the Upper-deck and Taking-off in the Lower-Deck

This technique is called for when a number on a specific rod is smaller than the supposed subtrahend (I have no idea what this means), but only in such cases as exemplified by 12 - 6.

TABLE 8 Combined Taking-off...

Original Number	Formula	Bead-Operations Involved		
		lower-deck, 1 rod right	lower-deck	upper-deck
1,2,3 or 4	minus 6 (-1+5-10)	-1	-1	+1
2,3 or 4	minus 7 (-2+5-10)	-1	- 2	+1
3 or 4	minus 8 (-3+5-10)	-1	- 3	+1
4	minus9 (-4+5-10)	-1	- 4	+1

If you've actually read this user's guide up to this point, hoping to learn how to use the abacus to its maximum potential, I should tell you that you'd be better off using a hand-held (or pop-up) digital calculator. (I hear you asking: Why did I bother going through all this if I was going to expound this heresy at the conclusion? Essentially, the exercise was designed to be didactic; from my programming stand-point, it was an interesting program to attempt and hopefully, the user would learn a thing or two. Unquestionably, though, it was unmitigated fun!). Technically, the abacus *is* a hand-held digital calculator, but that the user must perform some sort of manipulations before the solution is arrived at.

4.0 Acknowledgments

A great debt of gratitude goes to Agustine Lee, instructor at the Ryerson Electrical Engineering Department, who supplied a real, live abacus without which *xabacus* would not be possible, for supplying invaluable documentation, that was shamelessly plagiarized into the documentation you are now reading, for the Chinese characters, and for testing *xabacus* and providing helpful comments on improving it....

...thanks also to Nick Colonello, systems administrator and all-round technical-support person for beta testing....

...and to Eva Dudova, who has expertise in unmercifully crashing applications (has a future as a beta-tester), and is cute too...

... and finally, thanks to those who have written X-applications, from whose code I have learned the art of X.

This document was typeset using FrameMaker 2.1 (and then we got version 3.0 which would have really come-in handy for making all those tables) running on a SPARCstation 1.

5.0 Do those Chinese characters really mean anything?

A few[1] people have asked me this, so here's the translation:

The Abacus

The Calculation with the Abacus

Bead	*Calculate*	*Plate*
珠	算	盤

With respect to the arrangement in the top-left corner of every page, the characters may be read left-to-right or top-to-bottom.

1. A grand total of 1, actually.